

# **Rearranging equations**

#### A LEVEL LINKS

Scheme of work: 6a. Definition, differentiating polynomials, second derivatives

Textbook: Pure Year 1, 12.1 Gradients of curves

# **Key points**

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

# **Examples**

**Example 1** Make *t* the subject of the formula v = u + at.

v = u + at $v - u = at$	1 Get the terms containing <i>t</i> on one side and everything else on the other side.
$t = \frac{v - u}{a}$	<b>2</b> Divide throughout by <i>a</i> .

**Example 2** Make *t* the subject of the formula  $r = 2t - \pi t$ .

$r = 2t - \pi t$	<b>1</b> All the terms containing <i>t</i> are already on one side and everything else is on the other side.
$r = t(2 - \pi)$ $t = \frac{r}{2 - \pi}$	<ul> <li>2 Factorise as t is a common factor.</li> <li>3 Divide throughout by 2 - π.</li> </ul>

**Example 3** Make *t* the subject of the formula  $\frac{t+r}{5} = \frac{3t}{2}$ .

$\frac{t+r}{5} = \frac{3t}{2}$	1 Remove the fractions first by multiplying throughout by 10.
2t + 2r = 15t	2 Get the terms containing <i>t</i> on one side and everything else on the other
2r = 13t	side and simplify.
$t = \frac{2r}{13}$	<b>3</b> Divide throughout by 13.





#### Make t the subject of the formula $r = \frac{3t+5}{t-1}$ . Example 4

$$r = \frac{3t+5}{t-1}$$

$$r(t-1) = 3t+5$$

$$rt - r = 3t+5$$

$$rt - 3t = 5 + r$$

$$t(r-3) = 5 + r$$

$$r(t-1) = 3t + 5$$

$$rt - r = 3t + 5$$

$$rt - 3t = 5 + t$$

$$t(r-3) = 5 + r$$

$$t = \frac{5+r}{r-3}$$

- 1 Remove the fraction first by multiplying throughout by t-1.
- 2 Expand the brackets.
- 3 Get the terms containing t on one side and everything else on the other
- Factorise the LHS as t is a common
- 5 Divide throughout by r 3.

### **Practice**

Change the subject of each formula to the letter given in the brackets.

1 
$$C = \pi d$$
 [d]

2 
$$P = 2l + 2w$$
 [w]

$$3 D = \frac{S}{T} [T]$$

$$4 p = \frac{q-r}{t} [t]$$

**4** 
$$p = \frac{q-r}{t}$$
 [t] **5**  $u = at - \frac{1}{2}t$  [t] **6**  $V = ax + 4x$  [x]

**6** 
$$V = ax + 4x$$
 [x]

7 
$$\frac{y-7x}{2} = \frac{7-2y}{3}$$
 [y] 8  $x = \frac{2a-1}{3-a}$  [a] 9  $x = \frac{b-c}{d}$  [d]

8 
$$x = \frac{2a-1}{3-a}$$
 [a]

$$9 x = \frac{b-c}{d} [d]$$

**10** 
$$h = \frac{7g - 9}{2 + g}$$
 [g]

11 
$$e(9+x) = 2e+1$$
 [e

**11** 
$$e(9+x) = 2e+1$$
 [e] **12**  $y = \frac{2x+3}{4-x}$  [x]

Make *r* the subject of the following formulae.

$$\mathbf{a} \qquad A = \pi r^2$$

$$\mathbf{b} \qquad V = \frac{4}{3}\pi r^3$$

$$\mathbf{c} \qquad P = \pi r + 2$$

**a** 
$$A = \pi r^2$$
 **b**  $V = \frac{4}{3}\pi r^3$  **c**  $P = \pi r + 2r$  **d**  $V = \frac{2}{3}\pi r^2 h$ 

**14** Make *x* the subject of the following formulae.

$$\mathbf{a} \qquad \frac{xy}{z} = \frac{ab}{cd}$$

$$\mathbf{b} \qquad \frac{4\pi cx}{d} = \frac{3z}{py^2}$$

- Make sin B the subject of the formula  $\frac{a}{\sin A} = \frac{b}{\sin B}$
- Make  $\cos B$  the subject of the formula  $b^2 = a^2 + c^2 2ac \cos B$ .

# **Extend**

17 Make x the subject of the following equations.

$$\mathbf{a} \qquad \frac{p}{q}(sx+t) = x-1$$

$$\mathbf{b} \qquad \frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$$





## **Answers**

$$1 d = \frac{C}{\pi}$$

$$2 w = \frac{P - 2l}{2} 3 T = \frac{S}{D}$$

$$T = \frac{S}{L}$$

$$4 t = \frac{q-r}{p}$$

$$5 t = \frac{2u}{2a-1}$$

5 
$$t = \frac{2u}{2a-1}$$
 6  $x = \frac{V}{a+4}$ 

7 
$$y = 2 + 3x$$

$$8 \qquad a = \frac{3x+3}{x+2}$$

8 
$$a = \frac{3x+1}{x+2}$$
 9  $d = \frac{b-c}{x}$ 

**10** 
$$g = \frac{2h+9}{7-h}$$

11 
$$e = \frac{1}{x+7}$$

11 
$$e = \frac{1}{x+7}$$
 12  $x = \frac{4y-3}{2+y}$ 

**13 a** 
$$r = \sqrt{\frac{A}{\pi}}$$
 **b**  $r = \sqrt[3]{\frac{3V}{4\pi}}$ 

$$\mathbf{b} \qquad r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$\mathbf{c} \qquad r = \frac{P}{\pi + 2}$$

$$\mathbf{c} \qquad r = \frac{P}{\pi + 2} \qquad \qquad \mathbf{d} \qquad r = \sqrt{\frac{3V}{2\pi h}}$$

**14 a** 
$$x = \frac{abz}{cdy}$$

**14** a 
$$x = \frac{abz}{cdy}$$
 b  $x = \frac{3dz}{4\pi cpy^2}$ 

$$15 \quad \sin B = \frac{b \sin A}{a}$$

16 
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\mathbf{17} \quad \mathbf{a} \qquad x = \frac{q+pt}{q-ps}$$

17 **a** 
$$x = \frac{q+pt}{q-ps}$$
 **b**  $x = \frac{3py+2pqy}{3p-apq} = \frac{y(3+2q)}{3-aq}$