

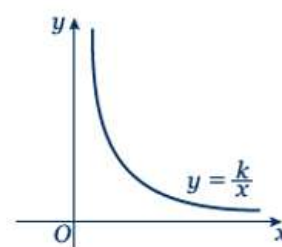
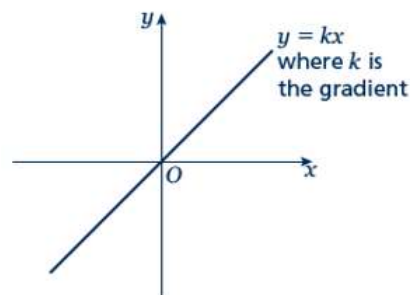
Proportion

A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

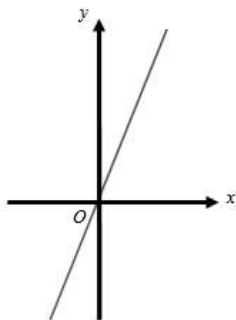
- Two quantities are in direct proportion when, as one quantity increases, the other increases at the same rate. Their ratio remains the same.
- ' y is directly proportional to x ' is written as $y \propto x$.
If $y \propto x$ then $y = kx$, where k is a constant.
- When x is directly proportional to y , the graph is a straight line passing through the origin.
- Two quantities are in inverse proportion when, as one quantity increases, the other decreases at the same rate.
- ' y is inversely proportional to x ' is written as $y \propto \frac{1}{x}$.
If $y \propto \frac{1}{x}$ then $y = \frac{k}{x}$, where k is a constant.
- When x is inversely proportional to y the graph is the same shape as the graph of $y = \frac{1}{x}$.



Examples

- Example 1** y is directly proportional to x .
When $y = 16$, $x = 5$.
- Find x when $y = 30$.
 - Sketch the graph of the formula.

a $y \propto x$ $y = kx$ $16 = k \times 5$ $k = 3.2$ $y = 3.2x$ When $y = 30$, $30 = 3.2 \times x$ $x = 9.375$	- Write y is directly proportional to x, using the symbol $\propto$. - Write the equation using k. - Substitute $y = 16$ and $x = 5$ into $y = kx$. - Solve the equation to find k. - Substitute the value of k back into the equation $y = kx$. - Substitute $y = 30$ into $y = 3.2x$ and solve to find x when $y = 30$.
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b 	7 The graph of $y = 3.2x$ is a straight line passing through $(0, 0)$ with a gradient of 3.2.
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Example 2 y is directly proportional to x^2 .
When $x = 3$, $y = 45$.

- a** Find y when $x = 5$.
b Find x when $y = 20$.

a $y \propto x^2$ $y = kx^2$ $45 = k \times 3^2$ $k = 5$ $y = 5x^2$ When $x = 5$, $y = 5 \times 5^2$ $y = 125$ b $20 = 5 \times x^2$ $x^2 = 4$ $x = \pm 2$	1 Write y is directly proportional to x^2, using the symbol $\propto$. 2 Write the equation using k. 3 Substitute $y = 45$ and $x = 3$ into $y = kx^2$. 4 Solve the equation to find k. 5 Substitute the value of k back into the equation $y = kx^2$. 6 Substitute $x = 5$ into $y = 5x^2$ and solve to find y when $x = 5$. 7 Substitute $y = 20$ into $y = 5x^2$ and solve to find x when $y = 20$.
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Example 3 P is inversely proportional to Q .
When $P = 100$, $Q = 10$.
Find Q when $P = 20$.

$P \propto \frac{1}{Q}$ $P = \frac{k}{Q}$ $100 = \frac{k}{10}$ $k = 1000$ $P = \frac{1000}{Q}$ $20 = \frac{1000}{Q}$ $Q = \frac{1000}{20} = 50$	1 Write P is inversely proportional to Q, using the symbol $\propto$. 2 Write the equation using k. 3 Substitute $P = 100$ and $Q = 10$. 4 Solve the equation to find k. 5 Substitute the value of k into $P = \frac{k}{Q}$. 6 Substitute $P = 20$ into $P = \frac{1000}{Q}$ and solve to find Q when $P = 20$.
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Practice

Hint

Substitute the values given for P and h into the formula to calculate k .

- 1 Paul gets paid an hourly rate. The amount of pay (£ P) is directly proportional to the number of hours (h) he works.
When he works 8 hours he is paid £56.
If Paul works for 11 hours, how much is he paid?
- 2 x is directly proportional to y .
 $x = 35$ when $y = 5$.
 - a Find a formula for x in terms of y .
 - b Sketch the graph of the formula.
 - c Find x when $y = 13$.
 - d Find y when $x = 63$.
- 3 Q is directly proportional to the square of Z .
 $Q = 48$ when $Z = 4$.
 - a Find a formula for Q in terms of Z .
 - b Sketch the graph of the formula.
 - c Find Q when $Z = 5$.
 - d Find Z when $Q = 300$.
- 4 y is directly proportional to the square of x .
 $x = 2$ when $y = 10$.
 - a Find a formula for y in terms of x .
 - b Sketch the graph of the formula.
 - c Find x when $y = 90$.
- 5 B is directly proportional to the square root of C .
 $C = 25$ when $B = 10$.
 - a Find B when $C = 64$.
 - b Find C when $B = 20$.
- 6 C is directly proportional to D .
 $C = 100$ when $D = 150$.
Find C when $D = 450$.
- 7 y is directly proportional to x .
 $x = 27$ when $y = 9$.
Find x when $y = 3.7$.
- 8 m is proportional to the cube of n .
 $m = 54$ when $n = 3$.
Find n when $m = 250$.

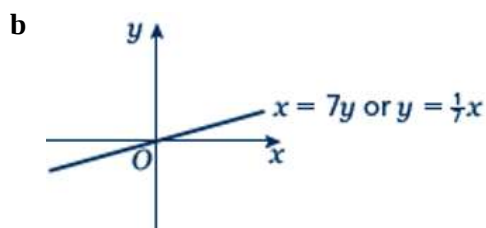
Extend

- 9 s is inversely proportional to t .
- a Given that $s = 2$ when $t = 2$, find a formula for s in terms of t .
 - b Sketch the graph of the formula.
 - c Find t when $s = 1$.
- 10 a is inversely proportional to b .
 $a = 5$ when $b = 20$.
- a Find a when $b = 50$.
 - b Find b when $a = 10$.
- 11 v is inversely proportional to w .
 $w = 4$ when $v = 20$.
- a Find a formula for v in terms of w .
 - b Sketch the graph of the formula.
 - c Find w when $v = 2$.
- 12 L is inversely proportional to W .
 $L = 12$ when $W = 3$.
Find W when $L = 6$.
- 13 s is inversely proportional to t .
 $s = 6$ when $t = 12$.
- a Find s when $t = 3$.
 - b Find t when $s = 18$.
- 14 y is inversely proportional to x^2 .
 $y = 4$ when $x = 2$.
Find y when $x = 4$.
- 15 y is inversely proportional to the square root of x .
 $x = 25$ when $y = 1$.
Find x when $y = 5$.
- 16 a is inversely proportional to b .
 $a = 0.05$ when $b = 4$.
- a Find a when $b = 2$.
 - b Find b when $a = 2$.

Answers

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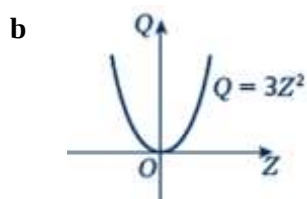
2 a $x = 7y$



c 91

d 9

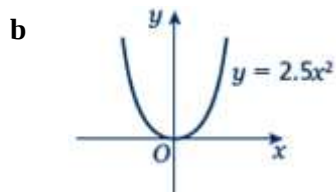
3 a $Q = 3Z^2$



c 75

d ± 10

4 a $y = 2.5x^2$



c ± 6

5 a 16

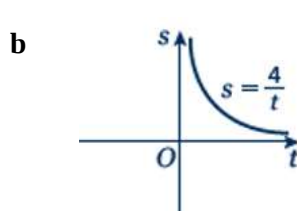
b 100

6 300

7 11.1

8 5

9 a $s = \frac{4}{t}$

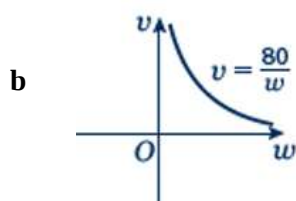


c 4

10 a 2

b 10

11 a $v = \frac{80}{w}$



c 40

12 6

13 a 24

b 4

14 1

15 1

16 a 0.1

b 0.1